On the Shaking Force Balancing of Compliant Mechanisms

S. Martínez[1], J. P. Meijaard[2], and V. van der Wijk[2]

Abstract— **This paper is a first approach in finding design principles for the design of shaking force balanced compliant mechanisms. Shaking force balance means that the motions of the mechanism do not create any resultant dynamic reaction forces on the base, eliminating base vibrations.**

It is found that for a single balanced rotatable flexible link two stiffness related balance conditions exist in addition to the balance condition known for a rigid link. With these conditions the shaking force balance of a planar parallelogram mechanism with flexible links is considered. The case with fully compliant hinges is applied to a planar translator and the results are compared with the case in which the hinges are real revolute joints. Simulations show perfect force balance for the model with revolute joints and a reduced shaking force of 67 % for the model with flexible joints. Prototypes of both mechanisms were developed and experimentally tested, showing shaking force reductions of 93 % and 97.5 %, respectively.

***Index Terms*— shaking force balance, compliant mechanism, flexible links, vibration reduction**

I. INTRODUCTION

Compliant mechanisms have various advantages over conventional rigid-body mechanisms, such as part-count reduction, reduced assembly time, simplified manufacturing processes, high repeatability, increased precision, lack of friction and backlash, reduced wear and no need for lubrication, among others [1], [2]. These benefits make them especially attractive for applications where precision and accuracy are needed, such as in positional stages [3] or motion transmissions.

Because of these advantages, the study and application of compliant mechanisms has increased in recent years in a variety of ways. Most of the research is focused on the structural and kinematic analysis and design of compliant mechanisms. However, the study of dynamic balancing of compliant mechanisms still is very limited, if existing. In the related field of rotor balancing a considerable amount of work has been done [4], however the results are not directly applicable to articulated compliant mechanisms. The force balance of a four-bar mechanism with a flexible coupler was studied in [5].

Dynamic balance consists of shaking force balance and shaking moment balance. A dynamically balanced mechanism does not exert any resultant reaction forces and resultant reaction moments to its base, preventing base vibrations.

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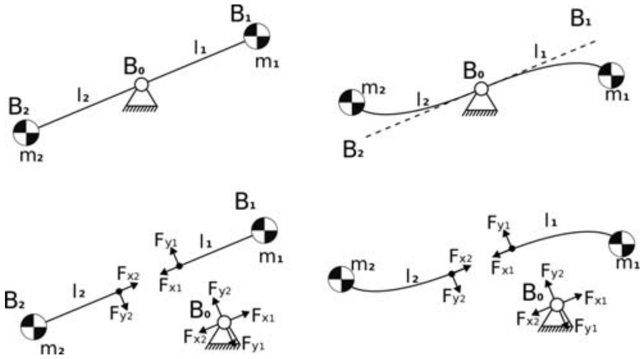

Fig. 1. Dynamic reaction forces on the base pivot B_0 of a rigid balanced single link (left side) and a flexible balanced single beam (right side).

Dynamic balance of mechanisms consisting of rigid bodies interconnected by ideal joints has been a subject in the study of machine dynamics for many years [6], [7], [8]. For shaking force balance the linear momentum of the moving masses needs to be constant (e.g. zero) for all motions. A zero linear momentum implies that the common center of mass (common CoM) of the mechanism is stationary for all motions. Similarly, for moment balancing the angular momentum of the mechanism needs to be constant (e.g. zero) for all motions [9].

The goal of this work is to investigate the shaking force balancing of linkages composed of single flexible links to find new balance principles for compliant mechanisms. The force balance of a single flexible link is studied first. Subsequently it is investigated how, based on this balanced flexible link, force balanced compliant parallelogram mechanisms can be obtained. This is shown for both situations of having flexible links with revolute pairs and having flexible links in a monolithic design with flexible joints. Of both parallelogram mechanisms a prototype was designed and tested in an experimental set-up. The measurements are compared with the calculated values from simulations.

II. THEORY AND DESIGN

In this section first the shaking force balance of a single flexible link is considered, followed by the designs of two parallelogram mechanisms each composed of two of these force balanced flexible links.

A. Single shaking force balanced flexible beam

Shaking force balancing has been well defined for a rigid body rotating link, however for a flexible beam it introduces new problems which have not been considered yet. The compliance of a beam means that when the beam is in

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motion, the body suffers from deformations which generally result into a non-stationary CoM of the beam, as well as oscillations that result in shaking forces. In Fig. 1 it is shown on the left how a rigid shaking force balanced link B_1B_2 with point masses m_1 and m_2 in B_1 and B_2, respectively, rotates about a base pivot B_0 with its exerted reaction forces F_{x1}, F_{x2}, F_{y1}, and F_{y2} to the base pivot. The motion and reaction forces of a similar flexible force balanced beam is illustrated on the right side.

For force balance the sum of the reaction forces at the base pivot must be zero, meaning that $F_{x1} = F_{x2}$ and $F_{y1} = F_{y2}$. The CoM therefore must be exactly located in the base pivot for all motions. For the rigid beam this means that the necessary condition for force balance can be written as

$$m_1 l_1 + \frac{1}{2} l_1^2 \rho_1 A_1 = m_2 l_2 + \frac{1}{2} l_2^2 \rho_2 A_2 \quad (1)$$

of which the parameters are explained in Table I. Generally m_2 is seen as the balancing mass of point mass m_1. The second terms on each side relate to the equally distributed masses of the beams.

This balance condition also holds for the flexible beam, however the CoM of the flexible beam could still oscillate following the pattern defined by its stiffness/mass relationship. To have $F_{x1} = F_{x2}$ and $F_{y1} = F_{y2}$, both sides of the beam need to oscillate in a similar way. From the relation for equal eigenfrequencies of both sides, two balance conditions for this can be found, written as

$$\sqrt{\frac{3E_1I_1}{m_1 l_1^3}} = \sqrt{\frac{3E_2I_2}{m_2 l_2^3}}, \quad \frac{1}{2} l_1^2 \rho_1 A_1 = \frac{1}{2} l_2^2 \rho_2 A_2 \quad (2)$$

This means that the three shaking force balance conditions for a single flexible beam result into

$$m_1 l_1 = m_2 l_2, \quad l_1^2 \rho_1 A_1 = l_2^2 \rho_2 A_2, \quad \frac{E_1 I_1}{l_1^2} = \frac{E_2 I_2}{l_2^2} \quad (3)$$

These conditions balance all skew-symmetric modes even in the non-linear regime, as long as the assumptions of beam theory and neglecting the size of the point masses remain valid. The symmetric modes are not force-balanced, so if these are excited by perturbations or by the initial conditions, the force balance is lost. It is interesting that these conditions imply that the longer one side of the beam is, the stiffer it needs to be.

TABLE I

DESIGN VALUES OF THE TWO COMPLIANT PARALLELOGRAM MODELS

Parameter	Meaning	Revolute joints	Flexible joints
m_1	tip mass	21.55 g	10.65 g
l_1	beam length	20.4 mm	54 mm
$\rho_1 A_1$	beam mass/lenght	0.064 g/mm	0.016 g/mm
$E_1 I_1$	flexural rigidity	533.33 Nmm^2	2133.33 Nmm^2
m_2	end mass	10.77 g	21.3 g
l_2	beam length	40.8 mm	27 mm
$\rho_2 A_2$	beam mass/lenght	0.016 g/mm	0.064 g/mm
$E_2 I_2$	flexural rigidity	2133.33 Nmm^2	533.33 Nmm^2

B. *Shaking force balanced compliant parallelogram linkages*

The shaking force balanced flexible beam can be used to design compliant parallelogram mechanisms. Figure 2 shows a parallelogram mechanism with revolute joints. Here the beams B_1B_2 and C_1C_2 are flexible while the translating bar B_1C_1 is rigid. This bar has twice the mass of m_1 which is uniformly distributed such that effectively there is a mass m_1 in each joint B_1 and C_1. The force balance conditions for each flexible beam (3) then determine the force balance of the complete mechanism.

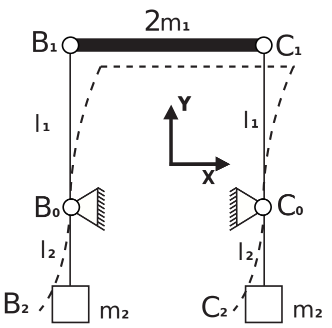

Fig. 2. Force balanced compliant parallelogram mechanism with two flexible beams and revolute joints.

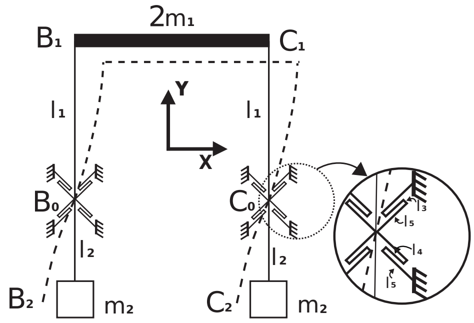

Fig. 3. Force balanced compliant parallelogram with two flexible beams and solely compliant joints. The base pivots consist of butterfly hinges.

Figure 3 shows a proposed solution of a shaking force balanced compliant parallel mechanism with solely compliant joints. By considering compliant base pivots a new problem is introduced: the necessity to find a flexible hinge suitable for balance, which has not been investigated before. For the purpose of this work an ideal flexible hinge for force balance should feature the following characteristics:

- A *low torsional stiffness* is desired to transmit as perfect as possible the beam deformations from one side to the other side of the joint, since joint stiffness causes undesired and unbalanced beam deformations.
- A *stationary center of rotation* is mandatory in order to achieve a stationary common CoM. A non-stationary center of rotation would result in motions of the common CoM, therefore leading to shaking forces.
- A *symmetrical mass distribution* is required. Some flexible hinges such as the cross-hinge flexure feature asymmetrical mass distributions which, when motion is applied, result in shaking forces.

The butterfly-like hinge [10] was selected as a potential best performing flexible hinge meeting these requirements and was implemented as base pivots in B_0 and C_0.

III. SIMULATIONS

In this section the models used for the simulations are presented. Also the obtained simulation results are discussed.

A. Simulation Models

The numerical model of the flexible beam and its dynamics were made with the flexible multibody dynamics program SPACAR [11]. The beam model used is the classical Euler–Bernoulli beam divided into 5 beam elements on each side of the fixed pivot with neglected elongation and shear deformation, and the mass distribution is assumed to be a line mass, so no moments of inertia of the cross-section are taken into account. First the model with revolute joints in Fig. 2 was created. The values in Table I, third column, were chosen which satisfy the force balance conditions (3). Then the model with flexible joints in Fig. 3 was made for the values in Table I, fourth column. The butterfly hinges as base pivots were designed with the values in Table II, which allow a proper tuning for low stiffness. The translating bar B_1C_1 has rigid connections with the flexible beams.

TABLE II
BUTTERFLY HINGE VALUES FOR PROTOTYPE MODEL

Parameter	Meaning	Value
$\rho_3 A_3$	Mass per unit of length	0.00064 g/mm
$E_3 I_3$	Flexural rigidity	853.33 Nmm^2
l_3	Length of leaf spring 3	1.75 mm
l_4	Length of leaf spring 4	9.3 mm
l_5	Length of leaf spring 5	19.3 mm

B. Simulation Results

After creating the models, the simulations were executed. Preloading the flexures is the method used to actuate the mechanism. This avoids the complexity of adding an actuator and it allows the visualization of the natural eigenfrequency. Fig. 4 shows the obtained resultant shaking force for the force balanced parallelogram with revolute joints. The results show negligible shaking forces in the frame, which are a consequence of the integrator error of the numerical calculations.

Figures 5 and 6 show the shaking forces obtained from the simulations of the parallelogram mechanism with flexible joints. Although the results show a reduction of, respectively, 67% along x and 76% along y of the force balanced case with respect to the unbalanced case with masses m_2 removed, the shaking forces are not equal to zero.

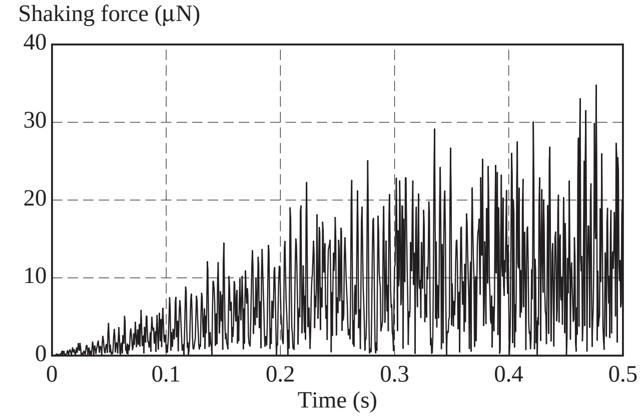

Fig. 4. Resultant shaking force from simulations of the force balanced mechanism with revolute joints in Fig. 2.

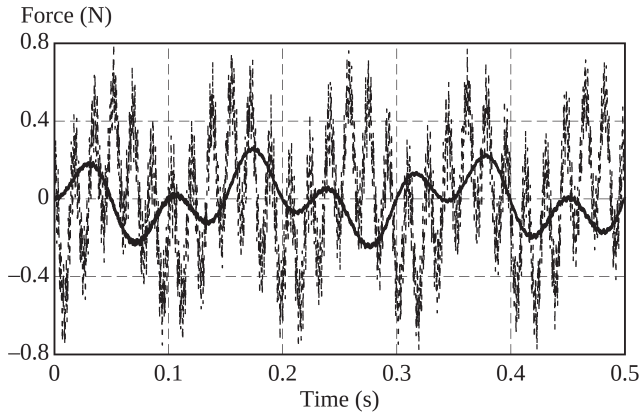

Fig. 5. Shaking forces in the x-direction from simulations of the unbalanced (dashed) and the force balanced (solid) mechanism with flexible joints in Fig. 3, showing a reduction of 67%.

IV. EXPERIMENTAL EVALUATION

In this section, the prototypes and the set-up used to test them are explained. The results obtained are presented and discussed.

A. Measurement Set-up

The two prototypes in Fig. 7 were built following the designs used for the simulations. They have been made out of laser cut acrylic plates for the rigid parts and the base frame, and of spring steel for the flexures. In order to actuate them a Lorentz actuator was used, which applies a force to the rigid translator and reacts against the base frame. To achieve consistent results the same force amplitude was used in all the tests. The moving part of the Lorentz actuator at the rigid bar is compensated with an additional mass on the other side to have the total mass of the bar exactly in the middle.

Since the mechanisms were designed to solely be shaking force balanced, they still have shaking moments. To be able to measure the shaking forces without the interference of the shaking moment, a specific set-up was created as shown in Fig. 8. The mechanism is placed with its white baseframe in a compliant translating stage which can freely float along the x-direction and constrains both the rotation and the motion in the y-direction. This means that only shaking forces in x-direction will cause translational motion of the stage.

A laser distance meter was used to measure the displacements of the floating stage at the location of the mirror. This measurement is considered proportional to the shaking force for a given mechanism excited at a given frequency. Table III shows the measurement devices used in this set-up.

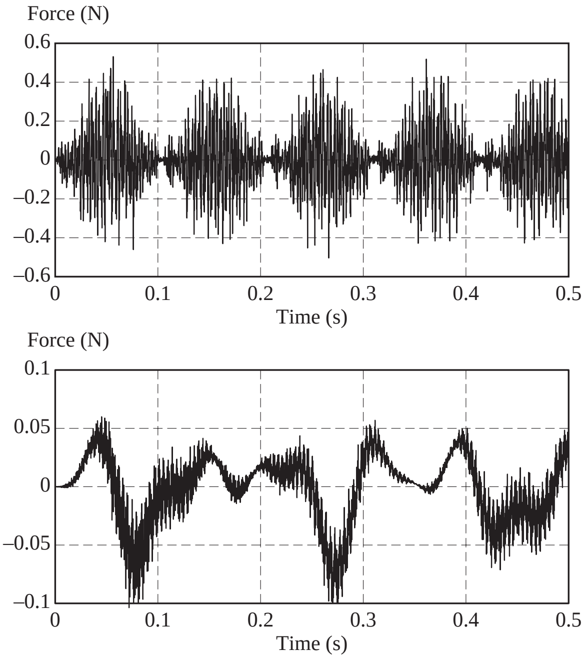

Fig. 6. Shaking forces in the y-direction from simulations of the unbalanced (top) and the force balanced (bottom) mechanism with flexible joints in Fig. 3, showing a reduction of 76%.

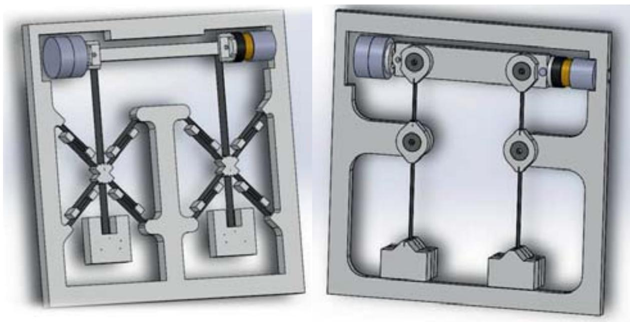

Fig. 7. CAD of prototype model with flexible joints (left) and of prototype model with revolute joints (right) with lorentz actuator in top right corner.

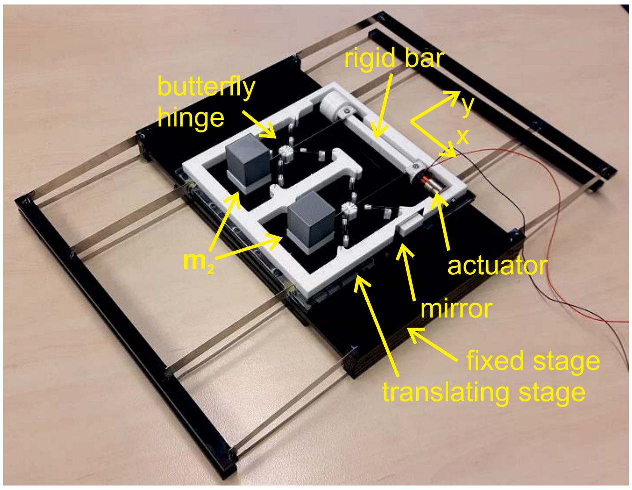

Fig. 8. Compliant translating stage for testing the prototypes which can float in a single direction along the x-axis. Shown with the prototype of the mechanism with flexible joints of Fig. 3.

TABLE III

MEASUREMENT DEVICES FOR EXPERIMENTS

Device	Model
Laser Sensor	optoNDC ild1420-10
Data Acquisition Board	National Instuments NI USB-6211

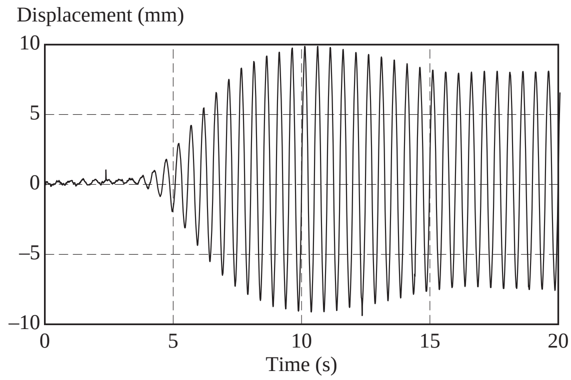

Fig. 9. Measured displacements of stage in x-direction at 2 Hz for the unbalanced compliant parallelogram with revolute joints.

B. Measurement Results

The measurements have been carried out for five different frequencies (1, 1.5, 2, 2.5, 3 Hz). The mechanism was stationary at the initial state, the actuation was increased gradually until the desired amplitude was achieved. Each test has been done with and without the balance masses m_2 in order to show the differences.

Fig. 9 shows the displacements measured when testing the prototype with revolute joints without the masses m_2 at 2 Hz. Comparing it with the data shown in Fig. 10 which corresponds to the force balanced case with masses m_2, we can observe a reduction by 93 %.

Fig. 11 shows the displacements measured when testing the prototype with compliant joints without the masses m_2 at 2 Hz. When comparing it with the data shown in Fig. 12 which corresponds to the force balanced case with masses m_2, we can observe a reduction by 97.5 %.

C. Discussion of Results

The results for the mechanism with revolute joints agree quite well with the qualitative predictions of the model. On the other hand, the measured reduction in shaking force for the mechanism with flexible joints is significantly higher than predicted by the model. This can be due to the different way of excitation and due to modeling the balance masses m_2 as point masses, whereas they have a non-negligible size in the prototype model. Also the effect of self-balancing due to oscillations of the two end-masses may play a role. With a more accurate model of the experimental conditions, a better agreement is expected. This is a subject of future research.

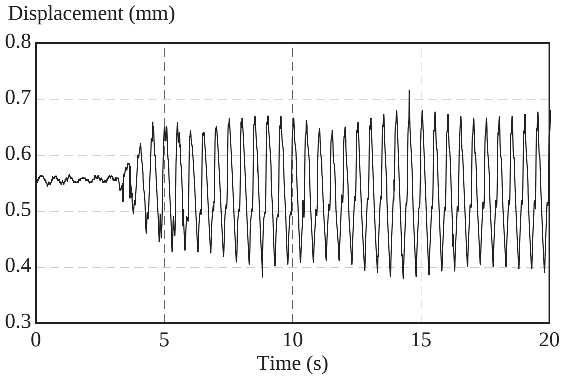

Fig. 10. Measured displacements of stage in x-direction at 2 Hz for the shaking force balanced compliant parallelogram with revolute joints, showing a reduction of 93% as compared to the unbalanced case.

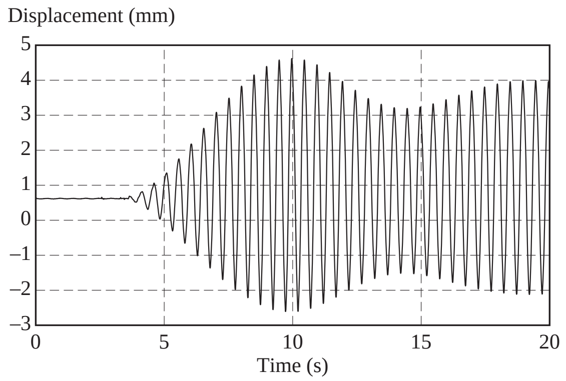

Fig. 11. Measured displacements of stage in x-direction at 2 Hz for the unbalanced compliant parallelogram with flexible joints.

V. CONCLUSIONS

In this paper the shaking force balancing of a flexible beam rotating about a single base pivot was investigated. It was found that in addition to the known force balance condition for a rigid beam rotating about a single base pivot, a second and a third force balance condition exist which relate the stiffness of the beam segments on each side of the base pivot. These conditions imply that the longer one side of the beam is, the stiffer it needs to be. Considering this as a design principle, the balanced flexible beam was used to design two compliant parallelogram mechanisms, consisting of a design with revolute joints and a design with solely flexible joints, which were presented.

The two parallelogram mechanisms were simulated with dynamic models and prototypes were developed. The simulations showed perfect force balance for the model with revolute joints and a reduced shaking force of 67% for the model with flexible joints. The prototypes were tested in an experimental set-up, showing a reduction of shaking force of 97.5 % for the parallelogram with flexible joints and a reduction of 93 % for the parallelogram with revolute joints, as compared to their unbalanced cases.

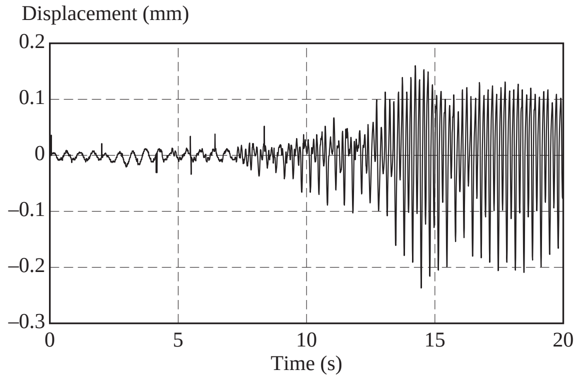

Fig. 12. Measured displacements of stage in x-direction at 2 Hz for the shaking force balanced compliant parallelogram with flexible joints, showing a reduction of 97.5% as compared to the unbalanced case.

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