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On a method for performance evaluation of the decoupled motion output of a non-linear parallel mechanism

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Abstract. Decoupled parallel mechanisms (PMs) are needed for devices with simpler and more intuitive adjustment capability. Currently, the design of fully-decoupled PMs is mostly based on linear structures. This paper considers decoupled PMs with a non-linear structure with locally decoupled outputs for a certain range. The goal of this paper is to present an evaluation method for the decoupling performance. As an example, the method is applied for the evaluation of the decoupled motion performance of a non-linear PM based on a planar 5R (five-bar) mechanism. In the method, sensitivity vectors that are based on velocity influence coefficients are used to investigate local decoupling conditions. A local cross-talk measure is proposed and applied for finding the best alignment of the mechanism for minimal total cross-talk for the given workspace.

Keywords. performance evaluation, decoupled motion, non-linear, parallel mechanism, cross-talk

1. Introduction

In the high-tech semiconductor industry, high precision and high speed are prominent concepts. Lithography machines depend on adjustable mounts which are used for alignment of optical components and inspection. Even though there are some motorized mechanisms, during installation the adjustments of the mounts are done manually [1]. Due to the coupled structure of the mounts, currently the adjustments cannot be done per one single degree of freedom (DOF) independently, making it a challenging task. Decoupled mounts therefore would be a significant improvement. A mechanism is called *fully decoupled* if each output parameter depends on only one single input parameter. As a result, they have simpler and more intuitive control [2]. In kinematic synthesis methods, decoupled mechanisms can be designed with serial or parallel architectures. When com-

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pared to the serial counterparts, parallel mechanisms (PMs) have better characteristics in terms of accuracy, stiffness and payload [3].

In literature, different types of fully decoupled PMs can be found. Kong and Gosselin proposed decoupled PMs which have 2T, 2R1T and 3T1R (T: translational, R: rotational) motions by using the leg-surface method [4]. Considerable work has been done on fully decoupled 3T-type PMs by different authors [5–9]. Gogu proposed many uncoupled PMs that vary from 2 to 4-DOF by using linear transformations [10–12]. In the design of those PMs, the most common way of obtaining decoupled motion is to use orthogonalities between legs and/or between actuators. Also, those structures are designed in such a way that the kinematic relation between one single input and output is independent from others. In other words, their structures are linear. The main advantages of this approach consists of having a modular synthesis procedure and having kinematically exact decoupled outputs without cross-talks. However, linear structures generally have some disadvantages like large volume and loss of rigidity due to poor payload distribution among the legs. On the other hand, non-linear structures can be another option for decoupled mechanisms. Such mechanisms are locally decoupled in certain configurations [13] and they can be used in the applications where relatively small workspaces are needed.

With a non-linear mechanism, the degree of coupling of the output of the mechanism is different in each point of workspace. In addition, structural parameters and workspace dimensions affect the overall amount of coupling. The decoupling is evaluated by a performance index which are mostly based on the condition number and is used to represent the dexterity of the mechanism and the closeness to a singularity in a specific pose [14]. When the condition number is equal to 1, the mechanism is called *isotropic* which means that the mechanism has identical kinetostatic properties in all the directions. However, in terms of decoupled motion performance, the condition number has no clear meaning [15].

The goal of this study is to propose an evaluation method for the decoupled motion performance of non-linear PMs. As an evaluation method, the sensitivity vectors based on the velocity influence coefficients of Jacobian matrix are to be used and local decoupling conditions are discussed. Then, a local cross-talk measure is proposed and applied to a pose to find the best alignment of the mechanism for the minimum total cross-talk for a given workspace. For this paper, a planar 5R five-bar mechanism is used as the case for a non-linear PM. Thus, this paper is limited to an evaluation method for a five-bar mechanism. Lastly, a general evaluation procedure for the five-bar mechanism is given.

In Section 2, the forward kinematic analysis of a five-bar mechanism is given. Section 3 presents the cross-talk measure formulation and the evaluation procedure for a five-bar mechanism. In Section 4, the performance evaluation is carried out for the five-bar mechanism and some discussions are made. Section 5 concludes the paper.

2. Forward Kinematic Analysis of a Five-bar Mechanism

A non-linear PM for decoupled motions, by definition, needs to have at least two different input-output relationships. Practically, this corresponds to a mechanism with at least two inputs and two outputs. A planar five-bar (5R) mechanism can be a suitable candidate for such a mechanism. It is a single-loop mechanism and two active joints as inputs are

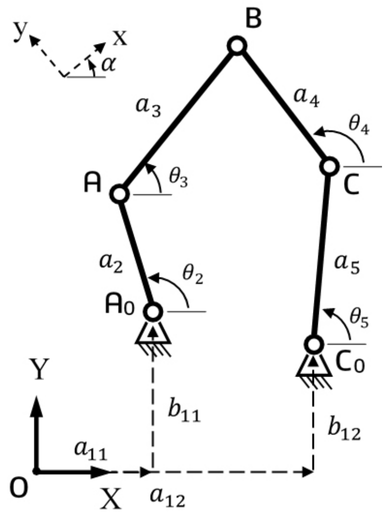


Figure 1. Kinematic diagram of a five-bar mechanism.

needed to control it for normally constrained control. The kinematic diagram of a five-bar mechanism is given in Figure 1. The rotations of the revolute joints in A_0 and C_0 are selected as the two input parameters and as the two output parameters the X and Y position of the joint in B are choosen.

In the forward kinematic analysis, the two input joint angles θ_2 and θ_5 are given, from which the position of joint B can be found. Analytical derivation of the direct equations can be accomplished by using the method of intersection of two circles [16], with which the position of point B is found by Eqs. (1) and (2). The forward kinematic singularities are presented in [17] and occur in the pose where links AB and BC become collinear.

$$X_B = X_A + d_x \frac{d_1}{d} - \frac{d_y}{d}\sqrt{{a_3}^2 - {d_1}^2} \tag{1}$$

$$Y_B = Y_A + d_y \frac{d_1}{d} + \frac{d_x}{d}\sqrt{{a_3}^2 - {d_1}^2} \tag{2}$$

where

$$\begin{aligned} X_A &= a_{11} + a_2 \cos\theta_2 & Y_A &= b_{11} + a_2 \sin\theta_2 \\ X_C &= a_{12} + a_5 \cos\theta_5 & Y_C &= b_{12} + a_5 \sin\theta_5 \\ d_x &= X_C - X_A & d &= \sqrt{{d_x}^2 + {d_y}^2} \\ d_y &= Y_C - Y_A & d_1 &= \frac{d^2 + {a_3}^2 - {a_4}^2}{2d} \end{aligned} \tag{3}$$

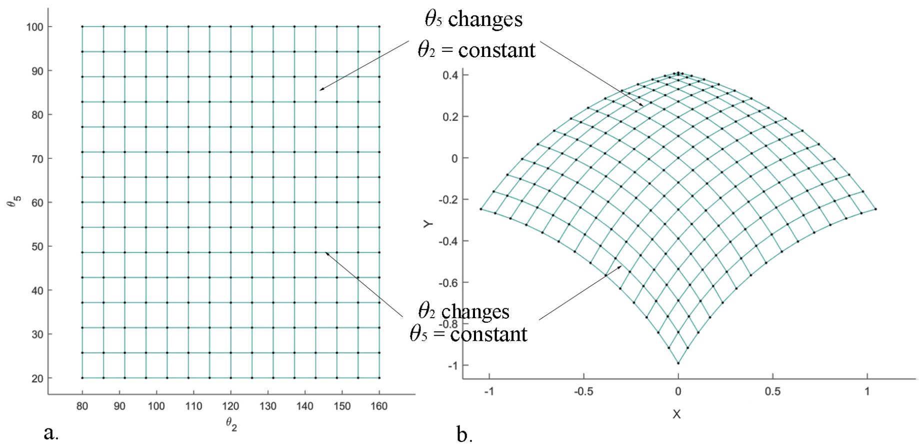


Figure 2. The position-level mapping from (a) the input space to (b) the output space of the five-bar mechanism with $a_{11} = -a_{12} = -0.50$, $b_{11} = b_{12} = -2.25$, $a_2 = a_5 = 1.00$, $a_3 = a_4 = 1.71$ and $\alpha = 0°$ for the input ranges: $80° < \theta_2 < 160°$ and $20° < \theta_5 < 100°$. The isolines show the effect of changing one input at a time with the curvature indicating the level of cross-talk.

The whole mechanism can be rotated by the angle α about O point to align the output space of the mechanism with the orthogonal XY-frame. In that case, the new coordinates of point B can be found as $x_B = X_B \cos\alpha - Y_B \sin\alpha$ and $y_B = X_B \sin\alpha + Y_B \cos\alpha$.

In Figure 2, an example of position-level mapping from input space to output space is given with discrete point sets and isolines for a five-bar mechanism. Without loss of generality, the length of the fixed link $|A_0C_0|$ is assumed 1 and the structural parameters are selected as: $a_{11} = -a_{12} = -0.50$, $b_{11} = b_{12} = -2.25$, $a_2 = a_5 = 1.00$, $a_3 = a_4 = 1.71$ and $\alpha = 0$. The input ranges are as follows: $80° < \theta_2 < 160°$ and $20° < \theta_5 < 100°$. In Figure 2.a, the input space is given as a square grid with equidistantly placed working points which define the pose of the mechanism. In this grid, the horizontal lines correspond to changing θ_2 while keeping θ_5 constant and it is visa-versa for the vertical lines. Figure 2.b shows the output space which is a warped grid. Moving along the isolines in the input space results in a curvilinear motion in the output space which is cause by the cross-talk effects. For a fully-decoupled mechanism without cross-talk effects, the isolines in the output space have to be straight as well and the lines for both motions must also be crossing each other perpendicularly throughout the workspace.

3. Cross-talk Formulation and the Evaluation Method for the Decoupled Output of a Five-bar Mechanism

Kineostatic analysis of parallel mechanisms depends on the Jacobian matrix which expresses the mapping between the input and output velocities. In Eq. (4), $\vec{\dot{\theta}}$ and $\vec{u}$ are the input and the output velocity vectors, respectively, and J is the Jacobian matrix which is a function of the input parameters. The jacobian matrix of the mechanism is given in Eq. (5). In this equation, $J_{11} = \partial x_B/\partial\theta_2$, $J_{12} = \partial x_B/\partial\theta_5$, $J_{21} = \partial y_B/\partial\theta_2$ and $J_{22} = \partial y_B/\partial\theta_5$ are the velocity influence coefficients. For a fully-decoupled mechanism, the Jacobian matrix is a diagonal matrix because the off-diagonal element: J_{12} and J_{21} define the cross-talks which are zero. For a five-bar mechanism, these terms are in general non-zero due to the fact that the mechanism is essentially coupled.

$$\vec{u} = J(\theta)\vec{\dot{\theta}} \tag{4}$$

$$\begin{bmatrix} \dot{x}_B \\ \dot{y}_B \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_5 \end{bmatrix} \tag{5}$$

$$\psi = \arccos\left(\frac{\vec{J}_{\theta_2} \cdot \vec{J}_{\theta_5}}{\left|\vec{J}_{\theta_2}\right| \left|\vec{J}_{\theta_5}\right|}\right) \tag{6}$$

$$K = \left(\frac{J_{21}}{J_{11}J_{22}}\right)^2 + \left(\frac{J_{12}}{J_{11}J_{22}}\right)^2 \tag{7}$$

It is known that the velocity influence coefficients can be defined as sensitivity vectors to represent the curvilinear isolines in the position-level output space. Therefore, for each workspace point, there are two sensitivity vectors which are related to the column vectors of the Jacobian matrix: $\vec{J}_{\theta_2} = [J_{11}\ J_{21}]^T$ and $\vec{J}_{\theta_5} = [J_{12}\ J_{22}]^T$. They give the information of how the output motion is changed by the input θ_2 and θ_5, respectively. Therefore, the effect of each input on the output motion can be investigated separately by using these sensitivity vectors. For an ideal decoupled output case, these vectors are perpendicular to each other and each perpendicular vector set is aligned with the orthogonal frame throughout the workspace [18]. To investigate that for a five-bar mechanism, the angle ψ between the two sensitivity vectors is calculated as in Eq. (6). In the specific points in the output space where ψ is equal or close to $90°$, the alignment of the sensitivity vectors with the orthogonal XY-frame can be accomplished by changing α. At that point, a cross-talk measure is needed to determine which alignment results in the minimum cross-talk. So, a local cross-talk measure based on the cross-talk terms J_{12} and J_{21} for a given α can be constructed. However, these cross-talk terms cannot reflect the effective level of cross-talk by themselves because J_{11} and J_{22} also change throughout the workspace. Therefore, the effect of the cross-talk terms can be measured if they are compared with the amount of output motion obtained as a result of given inputs. This comparison can be done in different ways depending on the task requirement. In Eq. (7), a local cross-talk measure is defined. In this equation, the cross-talk terms are normalized by the amount of output motion represented by $J_{11}J_{22}$. Also, two cross-talk terms are squared assuming that negative and positive cross-talks are equally unwanted. When K increases, the deviations of the isolines from the orthogonal axes are also increased.

For the evaluation procedure for the given five-bar mechanism, firstly, the sensitivity vector map is obtained for a large output range to get an overall idea and to detect the regions where $\psi = 90°$. Then, one of the poses corresponding to those regions is used as the initial pose for another investigation of a small output range to be used as the workspace of the mechanism. For the latter investigation, for different angles α, the

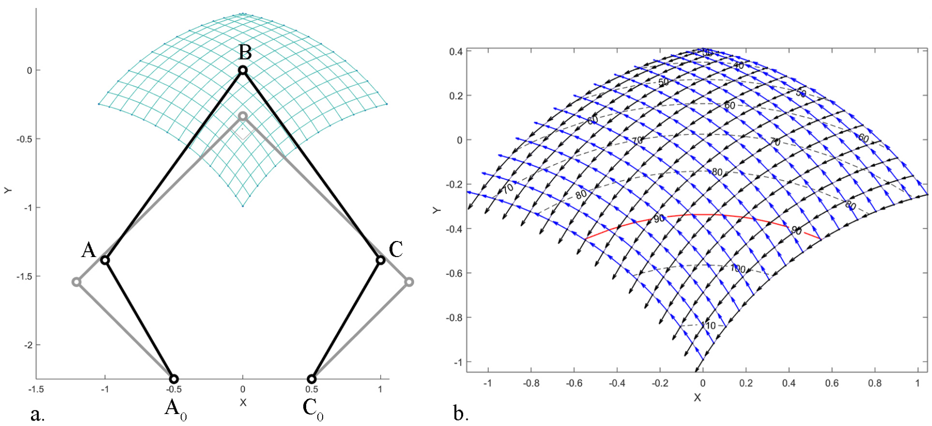


Figure 3. (a) The five-bar mechanism (in black) with $a_{11} = -a_{12} = -0.50$, $b_{11} = b_{12} = -2.25$, $a_2 = a_5 = 1.00$, $a_3 = a_4 = 1.71$ and $\alpha = 0°$ at home position. The mechanism in grey shows the pose when $\angle ABC = 90°$ and (b) The corresponding sensitivity vector map with the contour ψ for the input ranges: $80° < \theta_2 < 160°$ and $20° < \theta_5 < 100°$.

values of the local cross-talk are calculated by Eq. (7) for each workspace point and then they are summed to find the total value of cross-talk for the given workspace. Finally, the minimum total cross-talk amount and the corresponding alignment of the mechanism are found for a specific α.

4. Performance Evaluation for the Decoupled Output of a Five-bar Mechanism

For overall evaluation, the same input intervals are used as in Section 2. The mechanism is depicted in Figure 3.a at home position. The output space of this mechanism is given in Figure 3.b together with the corresponding sensitivity vector map. The black and the blue arrows represent $\vec{J}_{\theta_2}$ and $\vec{J}_{\theta_5}$, respectively. The dashed contour lines show ψ. As can be seen, ψ increases from 30° to 115° as going from top to bottom. The red curve shows where ψ is 90°. As going from left to right, the sensitivity vector sets rotate counter-clockwise. Also, the more ψ deviates from 90°, the curvier the isolines become. For a five-bar mechanism, it can be easily seen that ψ is directly related to the angle between links AB and BC. The red contour also shows the output locations where $\angle ABC = 90°$. In other words, when AB and BC links are perpendicular to each other, the sensitivity vectors are also perpendicular to each other as well. Hence, the home position of the mechanism should be selected on the red curve to have the most straight isolines. So, the pose corresponding to the middle point of the red curve is chosen as the new home position, which is illustrated in gray in Figure 3.a. The corresponding input parameters for this pose are $\theta_2 = 135°$ and $\theta_5 = 45°$.

To investigate the new pose in detail, smaller input intervals are chosen. In this case, the the input ranges are given as $125° < \theta_2 < 145°$ and $35° < \theta_5 < 55°$. The specific workspace with its sensitivity vector map are given in Figure 4. As can be seen, the isolines are almost straight since ψ deviates less from 90°. Also, they are approximately 45° angled with respect to the orthogonal xy frame. In Figure 5.a, the corresponding

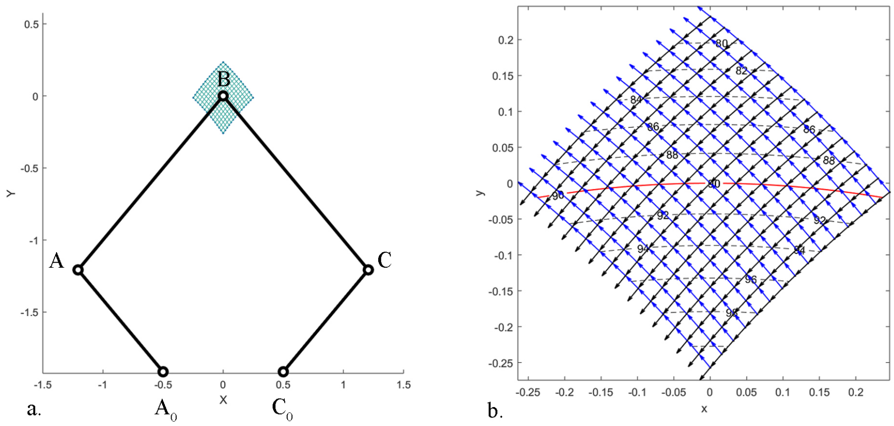


Figure 4. (a) The five-bar mechanism with $a_{11} = -a_{12} = -0.50$, $b_{11} = b_{12} = -1.91$, $a_2 = a_5 = 1.00$, $a_3 = a_4 = 1.71$ and $\alpha = 0°$ at home position and (b) The corresponding sensitivity vector map with ψ contour lines for the input ranges: $125° < \theta_2 < 145°$ and $35° < \theta_5 < 55°$ showing that the isolines are significantly less curvy.

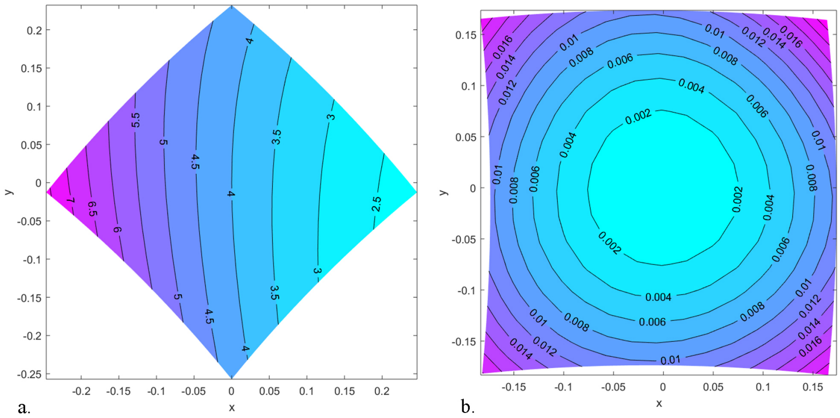
normalized cross-talk contours for $\alpha = 0°$ are given. In that figure, the local cross-talks increase as moving from left to right because both sensitivity vectors rotate counter-clockwise. That results in bigger cross-talk terms and smaller transmission ratio terms for both vectors. As moving from bottom to top, both vectors are getting more aligned with the horizontal axis. This results in smaller $\vec{J}_{\theta_2}$ and bigger $\vec{J}_{\theta_5}$. Due to the combined effect, the local cross-talks do not change much in the vertical direction.

When the local cross-talks are summed for the workspace, as α is changed from 0° to -45°, the total cross-talk amount is decreased from 951 to 1.82 and it is minimum when $\alpha = -45°$ as expected (see Figure 5.b). In this case, the best alignment of the black and the blue vectors with horizontal and vertical axes is achieved.

The total cross-talk amount for a five-bar mechanism is smaller when the workspace is smaller for the same link lengths or when the link lengths are larger for the same workspace dimensions. To compare the five-bar mechanisms with different link lengths in terms of decoupled output performance, fixed workspace dimensions should be used. Then, for each five-bar mechanism with different link lengths, the evaluation procedure presented in this paper can be applied to all possible poses along $\psi = 90°$ contour until the minimum total cross-talk amount is obtained. Thus, the optimum design of which the total cross-talk amount is acceptable for the task can be found.

5. Conclusion

This paper presented a method for the performance evaluation of the decoupled motion output of a non-linear parallel mechanism. In the evaluation, the sensitivity vectors based on the velocity influence coefficients were used to find the conditions for a decoupled output. Then, a local cross-talk measure was proposed. A planar 5R five-bar mechanism is evaluated as a case of non-linear PM. The method was applied to a pose to find the best alignment of the mechanism for the minimum total cross-talk for a given workspace. It was shown that the five-bar mechanism has relatively small cross-talks in that pose



a. b.

Figure 5. The normalized cross-talk contours of the five-bar mechanism for $125° < \theta_2 < 145°$ and $35° < \theta_5 < 55°$ when (a) $\alpha = 0°$ and (b) $\alpha = -45°$

and for that alignment. The method can also be applied for the comparison of different five-bar mechanisms.

To extend the evaluation method to other non-linear decoupled PM, the conditions for calculating cross-talk need to be extended to cover the cases where the sensitivity vectors are perpendicular to each other but the isolines in the output space are still curved. Also, in case the inputs and outputs consists of both rotations and translations, inhomogeneity in units would be an issue. A possible solution is to divide the elements of Jacobian matrix by a characteristic length. These issues will be investigated as future work.

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